

Neutron Star Properties with Hyperons

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In the light of the recent discovery of a neutron star with a mass accurately determined to be almost two solar masses, we carefully examine the maximum mass of a neutron star containing hyperons within a relativistic Hartree-Fock treatment. Using the quark-meson coupling model, which naturally incorporates hyperons without additional parameters, we find a maximum mass a little over $2.1 M_\odot$.

Keywords: neutron stars, equation of state of dense matter, hyperons, quarks

The recent observation of a $1.97 \pm 0.04 M_\odot$ millisecond pulsar, PSR J1614-2230, by Demorest *et al.* [1] has set the most stringent limit on models of neutron star cores so far. This discovery has spurred a re-examination of the possibility of exotica such as hyperons, Bose condensates, and quark matter playing an important role in models of neutron star interiors, owing to a presumed softening of the equation of state (EoS) expected in the presence of additional degrees of freedom. Historically, this has led to expectations of reduced maximum neutron star masses for compact objects in hydrostatic equilibrium.

In this Letter we build on the earlier work of Stone *et al.* [2], who already predicted the existence of neutron stars containing hyperons with masses as large as $2 M_\odot$ in 2007, to establish a conservative upper limit on the maximum mass of such a star. We work within the quark-meson coupling (QMC) model [3–5], which has the advantage of being derived from the quark level, with a very small number of adjustable parameters, while being consistent with a broad range of constraints derived from hypernuclei as well as normal nuclear properties. We find that the stability under variation of the very small number of adjustable parameters is such that if a star were discovered with a mass significantly above $2.1 M_\odot$, we would need to consider more exotic physics, because it could not be accommodated within the QMC model.

We recall that QMC is based upon the self-consistent modification of the structure of a baryon embedded in nuclear matter. At Hartree level it involves only three adjustable parameters which describe the effective couplings of the σ , ω and ρ mesons to the u and d quarks. These are fixed by adjusting them to fit the properties of symmetric nuclear matter, namely its saturation density and binding energy as well as its symmetry energy. We note that the σ meson used here simply serves as a convenient representation of the scalar-isoscalar attraction arising from two-pion exchange.

In the most recent development of the QMC model [5], the self-consistent inclusion of the gluonic hyperfine interaction led to a very successful description of the binding

energies of Λ -hypernuclei—as well as the observed absence of medium and heavy mass Σ -hypernuclei—with no additional parameters. We stress that this is achieved without any coupling of the strange quark to the σ , ω and ρ mesons (which would be OZI suppressed) and without the need to introduce any further mesons. While the model could be supplemented with much heavier mesons containing strange quarks [6], Occam’s razor suggests that one should not introduce them if they are not needed.

A clear connection has been established between the self-consistent treatment of in-medium hadron structure and the existence of many-body [7] or density dependent [8] effective forces. Dutra *et al.* [9] critically examined a variety of phenomenological Skyrme models of the effective density dependent nuclear force against the most up-to-date empirical constraints. Amongst the few percent of the Skyrme forces studied which satisfied all of these constraints, the Skyrme model SQMC700, derived from the QMC model, was unique in that it incorporated the effects of the internal structure of the nucleon and its modification in-medium.

While the earlier study of Stone *et al.* [2] demonstrated the importance of exchange (Fock) terms in calculations of the EoS of dense baryonic matter in β -equilibrium, it included only the Dirac vector term in the vector-meson-nucleon vertices. In this work we include the full vertex structure which one might expect to enhance the pressure at high density. This is especially so in the case of the ρ meson for which the tensor coupling is much larger than that of the ω . Our calculation extends the work of Krein *et al.* [10], who considered nucleons only, by evaluating the full exchange terms for all octet baryons and adding them in the same way as Stone *et al.* [2]; as additional contributions to the energy density. It also extends and complements the important work of Miyatsu *et al.* who calculated the properties of neutron stars within an earlier version of the QMC model [14]. In particular, we employ the latest version of the model which reproduces key hypernuclear properties without the adjustment of

coupling constants needed there. Furthermore, we carefully explore the limit on the maximum mass of a neutron star containing hyperons while ensuring consistency with critical nuclear properties, such as the incompressibility of nuclear matter.

Next we describe the main features of the QMC model and explore its parameters arranged into groups (“scenarios”) to test the robustness of its predictions. Within the QMC model, the baryon energy density, ϵ_B , is given by

$$\epsilon_B = \frac{2}{(2\pi)^3} \sum_B \int_{|\mathbf{p}| < p_F} d\mathbf{p} \sqrt{p^2 + M_B^{*2}}, \quad (1)$$

where the effective, in-medium baryon masses, M_B^* , are calculated self-consistently for an MIT bag immersed in (and in Ref. [5], parameterized as functions of) a mean scalar field, designated here by a barred symbol. At a given density, $\bar{\sigma}$ is self-consistently expressed as

$$\bar{\sigma} = -\frac{2}{m_\sigma^2 (2\pi)^3} \sum_B \int_{|\mathbf{p}| < p_F} d\mathbf{p} \frac{M_B^*}{\sqrt{p^2 + M_B^{*2}}} \frac{\partial M_B^*}{\partial \bar{\sigma}}. \quad (2)$$

An additional contribution, $\delta\bar{\sigma}$, to the scalar field arises if we include the Fock terms in the minimization of the energy density. This provides only a small correction to the mean field, and its effect is included only in the scenario denoted $\delta\bar{\sigma}$. The total hadronic energy density, ϵ_H , is the sum of baryonic, ϵ_B , and mesonic, $\epsilon_{\sigma\omega\rho\pi}$ contributions, for which

$$\begin{aligned} \epsilon_{\sigma\omega\rho\pi} = & \sum_{\alpha=\sigma,\omega,\rho} \frac{1}{2} m_\alpha^2 \bar{\alpha}^2 \\ & + \sum_{\alpha=\sigma,\omega,\rho,\pi} \sum_{BB'} \frac{C_{BB'}^\alpha}{(2\pi)^6} \iint_{\substack{|\mathbf{p}| < p_F \\ |\mathbf{p}'| < p_{F'}}} d\mathbf{p} d\mathbf{p}' \Xi_{BB'}^\alpha, \end{aligned} \quad (3)$$

where $C_{BB'}^\sigma = C_{BB'}^\omega = \delta_{BB'}$. $C_{BB'}^\rho$ and $C_{BB'}^\pi$, which arise from symmetry considerations, are given in Ref. [2]. Note that the π meson only contributes to the second term in Eq. (3) as it is coupled via a pseudo-vector current. For $\epsilon_{\sigma\omega\rho}$, the integrand has the form

$$\Xi_{BB'}^\alpha = \frac{1}{2} \sum_{s,s'} |\bar{u}_{B'}(p', s') \Gamma_\alpha u_B(p, s)|^2 \Delta_\alpha(\mathbf{k}), \quad (4)$$

where $\Delta_\alpha(\mathbf{k})$ is the Yukawa propagator for meson α with momentum $\mathbf{k} = \mathbf{p} - \mathbf{p}'$. For the vector mesons, the full vertex structure is included in the manner of Ref. [10] as

$$\Gamma_{\alpha B} = \epsilon_\mu^\alpha \left[g_{\alpha B} \gamma_\mu F_1^\alpha(k^2) + \frac{i f_{\alpha B} \sigma_{\mu\nu} k^\nu F_2^\alpha(k^2)}{2M_B^*} \right]. \quad (5)$$

As usual, the effect of short distance repulsion on the Fock terms is simulated by the replacement

$$\frac{\vec{q}^2}{(\vec{q}^2 + m^2)} \rightarrow 1 - \frac{m^2}{(\vec{q}^2 + m^2)} \quad (6)$$

from which the unit term is subtracted, thus eliminating a δ -function. The form factors $F_{1,2}^\alpha$ all have the same

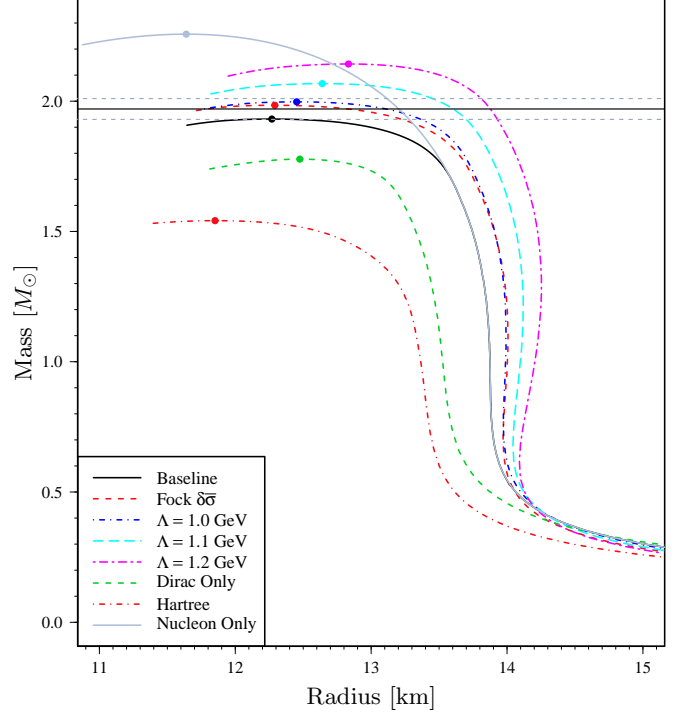


FIG. 1. Mass vs radius relation for a variety of scenarios described in the text. A BPS [17] crust has been added in all scenarios. We note that the maximum stable neutron star masses (indicated by filled circles) for essentially all full Hartree-Fock scenarios are compatible with the observations of Ref. [1] (denoted by a black horizontal line with grey dashed error-band).

dipole form with the cutoff mass Λ varied from 0.9 to 1.3 GeV to test the sensitivity. As a further test of the model dependence, we consider two choices for the ratios of tensor to vector coupling constants $\kappa_{\alpha B} = f_{\alpha B}/g_{\alpha B}$ ($\alpha \in \{\omega, \rho\}$). In scenario κ_I (consistent with values derived within QMC) we take these ratios from vector meson dominance ($\kappa_{\rho N} = f_{\rho N}/g_{\rho N} = 3.70$). Alternatively (scenario κ_{II}) we take these ratios from the Nijmegen potentials (Table VII of Ref. [11]), with a larger value of $\kappa_{\rho N} (= 5.7)$.

Of the baryon-meson coupling constants $g_{\sigma B}(\bar{\sigma})$, $g_{\omega B}$, and $g_{\rho B}$, only $g_{\sigma B}$ is density dependent. Its model parameterisation [5] is dependent on the free nucleon radius, which is taken to be $R_N^{\text{free}} = 1.0$ fm – with an alternate scenario having $R_N^{\text{free}} = 0.8$ fm. The density dependence is given by

$$g_{\sigma B}(\bar{\sigma}) = -\frac{\partial M_B^*}{\partial \bar{\sigma}} \equiv -\frac{\partial M_B^*(\bar{\sigma}, g_{\sigma N}, R_N^{\text{free}})}{\partial \bar{\sigma}}. \quad (7)$$

Values of the coupling constants $g_{\alpha N}$ for various mesons α and a selection of scenarios considered in this work are presented in Table I. The couplings $g_{\omega B}$ and $g_{\rho B}$ are expressed in terms of the quark level couplings

$$g_{\omega B} = n_{u,d}^B g_\omega^q; \quad g_{\rho B} = g_\rho N = g_\rho^q, \quad (8)$$

where $n_{u,d}^B$ represents the number of light quarks in baryon B . The σ , ω and ρ couplings to the quarks are

Scenario	$g_{\sigma N}$	$g_{\omega N}$	g_{ρ}	K (MeV)	R (km)	M_{\max} (M_{\odot})	n_c^{\max} (n_0)
κ_I	10.42	11.02	4.55	298	12.27	1.93	5.52
κ_{II}	10.55	11.09	3.36	299	12.19	1.93	5.62
$\Lambda = 1.0$	10.74	11.66	4.68	305	12.45	2.00	5.32
$\Lambda = 1.1$	11.10	12.33	4.84	312	12.64	2.07	5.12
$\Lambda = 1.2$	11.49	13.06	5.03	319	12.83	2.14	4.92
$\Lambda = 1.3$	11.93	13.85	5.24	329	13.02	2.23	4.74
$R = 0.8$	11.20	12.01	4.52	300	12.41	1.98	5.38
Fock $\delta\bar{\sigma}$	10.91	11.58	4.52	285	12.29	1.98	5.5
Dirac Only	10.12	9.25	7.83	294	12.56	1.79	5.2
Hartree Only	10.25	7.95	8.40	283	11.85	1.54	6.0
Nucleon Only	10.42	11.02	4.55	298	11.64	2.26	5.82

TABLE I. Meson-nucleon coupling constants determined for our baseline scenario ‘ κ_I ’ (for which $\Lambda = 0.9$ GeV, and $R_N^{\text{free}} = 1.0$ fm) and subsequent scenarios in which differences from κ_I are given in column 1. Also shown are the saturation incompressibility, K ; stellar radius; maximum stellar mass and corresponding central density (units $n_0 = 0.16 \text{ fm}^{-3}$).

constrained to reproduce a saturation energy per baryon of $\mathcal{E}_{\text{sat}} = -15.86$ MeV and an asymmetry energy coefficient of $a_{\text{asym}} = 32.5$ MeV at the saturation density $n_0 = 0.16 \text{ fm}^{-3}$. The ω , ρ and π masses are constrained to their experimental values, whereas the σ mass is taken to be 700 MeV.

For a compact object in β -equilibrium we solve the familiar system of equations for the number densities of the baryons and leptons [12]. The lepton energy density and pressure are given by the usual formulas for a degenerate Fermi gas. In order to obtain the neutron star properties shown in Table I, we solve the Tollmann–Oppenheimer–Volkov equations for the gravitational mass and radius [12]. The resulting dependence of the neutron star mass on radius, for a selection of the variations of the model, is shown in Fig. 1.

In Table I we present the coupling constants, incompressibilities of symmetric nuclear matter, and stellar properties, for a number of variations of the QMC model, in each scenario including the σ, π, ω and ρ Fock terms. We note that in all scenarios Ξ^- hyperons are present in significant quantities in the the maximum mass stars.

It is remarkable that in all of the scenarios investigated, the stellar properties are largely consistent, and similar to those reported by Stone *et al.* [2]. Scenarios in which the maximum stellar mass lies outside of the range 1.9–2.14 M_{\odot} correspond to nuclear matter compressibilities above the upper limit set in the recent comprehensive analysis of giant monopole resonance data [13]. While this cannot be true in general, it is certainly the case for the QMC model.

Turning to the effects of the inclusion of the full exchange terms on stellar properties, we find that the threshold density for Ξ^- is lowered, while those of Λ and

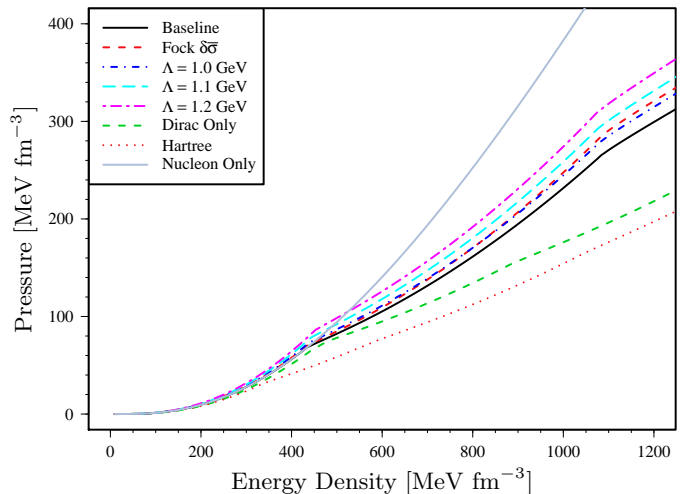


FIG. 2. Equation of state for a variety of scenarios. Kinks occur at significant hyperon threshold densities. The divergence between the baseline scenario (κ_I) and the ‘Hartree Only’ and ‘Dirac Only’ scenarios highlights the importance of the ρN tensor coupling in Hartree-Fock at high density.

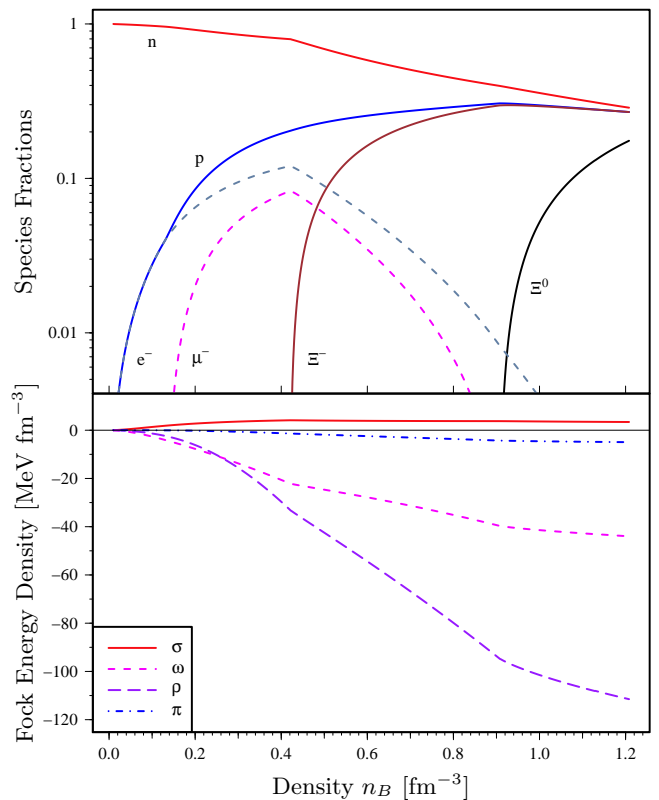


FIG. 3. Species fractions (upper) and Fock energy densities (lower) for our baseline (κ_I) scenario.

Ξ^0 are raised, as demonstrated in Fig. 3. In all scenarios there is a greater splitting between the thresholds of the Ξ baryons than that found by Stone *et al.* [2]. In our baseline scenario (κ_I), the Ξ^- threshold occurs at 0.42 fm^{-3} , followed by Ξ^0 at 0.91 fm^{-3} . We find that Λ

production is not energetically favoured at densities considered here, in agreement with Ref. [14]. Using the Nijmegen values of tensor coupling strength (κ_{II}), the ρN vector coupling is reduced as the tensor part of the interaction contributes significantly to the symmetry energy. The EoS is otherwise largely insensitive to this choice. Similarly, it is insensitive to the choice of free nucleon radius, despite a moderate impact on the couplings.

The correction ($\delta\bar{\sigma}$) to the scalar mean field arising from the Fock terms decreases the incompressibility by 13 MeV, yet other observables remain largely unaltered by this addition. The cutoff, Λ , used in form factors (which controls the strength of the Fock terms) exhibits a more pronounced relationship with the observables in Table I. Increasing Λ beyond 0.9 GeV raises the incompressibility with the case denoted $\Lambda = 1.3$ GeV already exceeding the limit of $K < 315$ MeV. We stress that Λ could not take a lower value without impacting the masses of the σ , ω , and ρ . The π , however, has a much lower mass and as such could involve a lower cutoff. We investigated this possibility but found only a minimal effect on the EoS, as expected from the small contribution the pion makes to the EoS at high density (refer to Fig. 3). Overall, increases in the cutoff correlate with increases in both the saturation incompressibility and maximum stellar mass.

We stress that the QMC model does not predict the appearance of Σ hyperons at any density where the model can be considered realistic. This is in contrast to a number of other relativistic models which do predict the Σ threshold to occur, even prior to that of the Λ [15, 16]. We note that Schaffner-Bielich [15] considered a phenomenological modification of the Σ potential with additional repulsion, which significantly raised its threshold density. In the case of the QMC model the physical explanation of the absence of Σ -hyperons is very natural, with the mean scalar field enhancing the repulsive hyperfine force in the bound Σ . Recall that the hyperfine splitting is due to one-gluon-exchange, which determines the free Σ - Λ mass splitting in the MIT bag model.

For comparison purposes, we also include a ‘Nucleon Only’ scenario, in which hyperons are artificially excluded. In this case the EoS is increasingly stiffer at densities above 0.4 fm^{-3} , leading to a large maximum stellar mass of $2.26 M_{\odot}$, consistent with many other nucleon-only models.

It is worth remarking that upon inclusion of the tensor coupling, the proton fraction increases more rapidly as a function of total baryon density. This is likely to increase the probability of the direct URCA cooling process in proto-neutron stars. As a further consequence, the maximum electron chemical potential is increased in this case, which may well influence the production of π^{-}

and \bar{K} condensates. Changes to the Λ threshold (occurs at higher density with lower maximum species fraction) reduces the possibility of H-dibaryon production as constrained by β -equilibrium of chemical potentials.

In summary, taking into account the full tensor structure of the vector-meson-baryon couplings in a Hartree-Fock treatment of the QMC model gives increased pressure at high density—largely because of the ρN tensor coupling—while maintaining reasonable values of incompressibility at saturation density. The conceptual separation between the incompressibility at saturation density and the amount of pressure or ‘stiffness’ at higher densities is critical. It is the latter that leads to neutron stars with maximum masses ranging from $1.90 M_{\odot}$ to $2.14 M_{\odot}$, even when allowance is made for the appearance of hyperons. This suggests that hyperons are very likely to play a vital role as constituents of neutron stars.

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